INDICES AND STANDARD FORM

Year 9

What do I need to be able to do?

Calculate powers and roots

Understand and use the index laws:

- Multiplying and dividing indices
- Power of a power
- Zero power
- Negative and fractional indices

Convert between standard form and ordinary numbers - large and small numbers

Perform calculations with numbers in standard form with and without a calculator

Keywords

Power: how we describe the calculation e.g. 4^3 is "4 to the power of 3"

Roots: the inverse of powers

Index number/ Indices: the small, raised number showing how many of the number will be multiplied together

Zero Power: anything to the power of zero e.g. a^0

Negative Indices: anything to the power of a number less than 0 e.g. a^{-3}

Fractional Indices: anything to the power of a fraction e.g. $a^{1/2}$

Standard Form: a method of writing very large or very small numbers

Calculate powers and roots:

Powers

$a^n = a \times a \times \ldots \times a$ "n" is the number of "a" multiplied

$a^2 = a \times a$ $a^3 = a \times a \times a$ and so on

$7^3 = 7 \times 7 \times 7 = 343$

Roots - $\sqrt{\ }$, $\sqrt[3]{\ }$ etc

These are the original numbers (the "a")

$\sqrt{\ }$ means square root and has an invisible 2 ($\sqrt[2]{\ }$)

$\sqrt{100} = 10$ because $10 \times 10 = 100$

$\sqrt[3]{\ }$ means cube root

$\sqrt[3]{64} = 4$ because $4 \times 4 \times 4 = 64$

Laws of Indices

Multiplying and Dividing Indices:

$a^m \times a^n = a^{m+n}$ example: $p^7 \times p^4 = p^{7+4} = p^{11}$

$a^m \div a^n = a^{m-n}$ example: $q^9 \div q^4 = q^{9-4} = q^5$

Mixed example:

$$\frac{(p^3 \times p^4 \times p^7)}{p^2 \times p^3} = \frac{p^{14}}{p^5} = p^9$$

Power of a power:

$(a^m)^n = a^{m \times n}$ example: $(p^3)^4 = p^3 \times p^4 = p^{12}$

Zero power:

$a^0 = 1$ Anything to the power of zero is always 1

Negative Powers:

$A^{-m} = \frac{1}{A^m}$ example: $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

Fractional Powers (numerator = 1):

$A^{1/n} = n^{th}$ root of $A = \sqrt[n]{A}$ example: $a^{1/4} = \sqrt[4]{a}$

Convert between standard form and ordinary numbers.

Standard form is $a \times 10^n$

"a" is a number between 1 and 10.

"n" is the number of times we multiply/divide by 10 to return to the ordinary number. Divide is shown as negative

ORDINARY NUMBERS TO STANDARD FORM

LARGE NUMBERS TO STANDARD FORM

Examples: Convert to standard form:

$27500000 = 2.75 \times 10^7$

Between 1 and 10   Number of digits after the 2

$450000 = 4.5 \times 10^5$

SMALL NUMBERS TO STANDARD FORM

Examples: Convert to standard form:

$0.00037 = 3.7 \times 10^{-4}$ Number of zeros in front of the 3

Between 1 and 10

$0.000000000251 = 2.51 \times 10^{-10}$

STANDARD FORM TO ORDINARY NUMBERS

Examples: Convert to ordinary numbers:

$8.1 \times 10^4 = 8.1 \times 10 \times 10 \times 10 \times 10 = 81000$

$4.67 \times 10^{-3} = 4.67 \div 10 \div 10 \div 10 = 0.00467$

Perform calculations with numbers in standard form with and without a calculator:

Addition / Subtraction

Convert to ordinary numbers and do the calculation.

Convert back to standard form

Example: $3.4 \times 10^4 + 2.7 \times 10^5$

$= 34000 + 270000$

$= 304000 = 3.04 \times 10^5$

Multiplication / Division	Example
Multiply the "a" values.	$(4.1 \times 10^3) \times (5 \times 10^5)$
Multiply the 10^n values.	$= (4.1 \times 5) \times (10^3 \times 10^5)$
Check if in Standard Form.	$= 20.5 \times 10^8$ (not S Form)
Adjust if necessary.	$= 2.05 \times 10^1 \times 10^8 =$ **2.05×10^9**

Higher Tier Topics Only

What do I need to be able to do?

- Simplify more complex algebraic or numerical expressions using several index laws.
- Simplify expressions where the base is not the same

Laws of Indices

Fractional powers (numerator >1):

$A^{m/n}$ = n^{th} root of a to the power of m

example: $a^{2/3} = (\sqrt[3]{a})^2$

$27^{2/3} = (\sqrt[3]{27})^2$

$= 3^2 = 9$

Simplify more complex algebraic/numerical expression using several index laws.

- $\frac{6a^3 \times 2ab^4}{3a^2b^2} = \frac{12a^4b^7}{3a^2b^2} = 4a^2b^5$
- $\frac{(p^2 \times p^{-7})^2}{p^8 \div p^3} = \frac{(p^{-5})^2}{p^5} = \frac{p^{-10}}{p^5} = p^{-2} = \frac{1}{p^2}$
- $\frac{2^3 \times 2^{-7} \times 2}{2^5} = \frac{2^{-3}}{2^5} = \frac{1}{2^5 \times 2^3} = \frac{1}{2^8} = \frac{1}{256}$

Simplify expressions where the base is not the same

- Simplify $\sqrt{81} \times 2^3 = 9 \times 8 = 72$
- Simplify $2^4 \times 3^{-2} = 16 \times \frac{1}{3^2} = 16 \times \frac{1}{9} = \frac{16}{9} = 1\frac{7}{9}$
- Write $8^{2/3} \times 16^{3/4}$ in the form 2^n

  $8^{2/3} = 2^2$ $16^{3/4} = 2^3$

  $2^2 \times 2^3 = 2^5$